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Universal computation and hydrodynamics

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Resumen

Is hydrodynamics capable of performing computations? or in other words, does there exist a fluid flow that can simulate any computer algorithm? This intriguing question was first formulated by C. Moore in 1991, and has been open for decades. In recent years, this problem has been revisited by T. Tao in relation with the celebrated blow-up problem in fluid mechanics: if such a universal "fluid computer" exists, it may be feasible to use it to design an initial datum that develops singularities when evolving with the Euler or Navier-Stokes equations. In this talk I will show how to construct stationary and time-dependent solutions of the Euler equations that are Turing complete, i.e., they can simulate any Turing machine. A striking consequence of these results is the existence of undecidable fluid particle paths, in the sense that there is no general algorithm to decide whether the trajectories of the flow starting at certain points will reach a certain (explicit) open set. This is a manifestation of complexity in hydrodynamics very different from the theory of chaos. This is based on the joint works [1, 2, 3, 4].

Referencias

[1] R. Cardona, E. Miranda, D. Peralta-Salas, F. Presas. *Universality of Euler flows and flexibility of Reeb embeddings*. Preprint.

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[3] R. Cardona, E. Miranda, D. Peralta-Salas. *Turing universality of the incompressible Euler equations and a conjecture of Moore*. Int. Math. Res. Notices, to appear.

[4] R. Cardona, E. Miranda, D. Peralta-Salas. *Computability and Beltrami fields in Euclidean space*. Preprint.